

Pushing the Precision Frontier of Higgs Production at the LHC

---- Three-Loop Higgs Plus Jet Amplitudes

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SLAC National Accelerator Laboratory

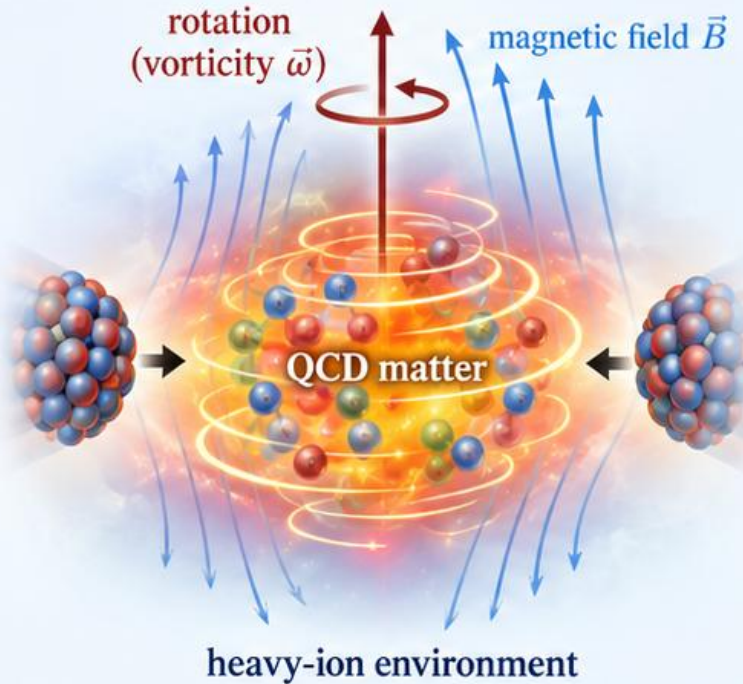
The 2nd Workshop on QCD Matter under Rotation and Magnetic Fields
13 September 2026



From QCD Matter under Rotation and Magnetic Fields to High-Energy QCD

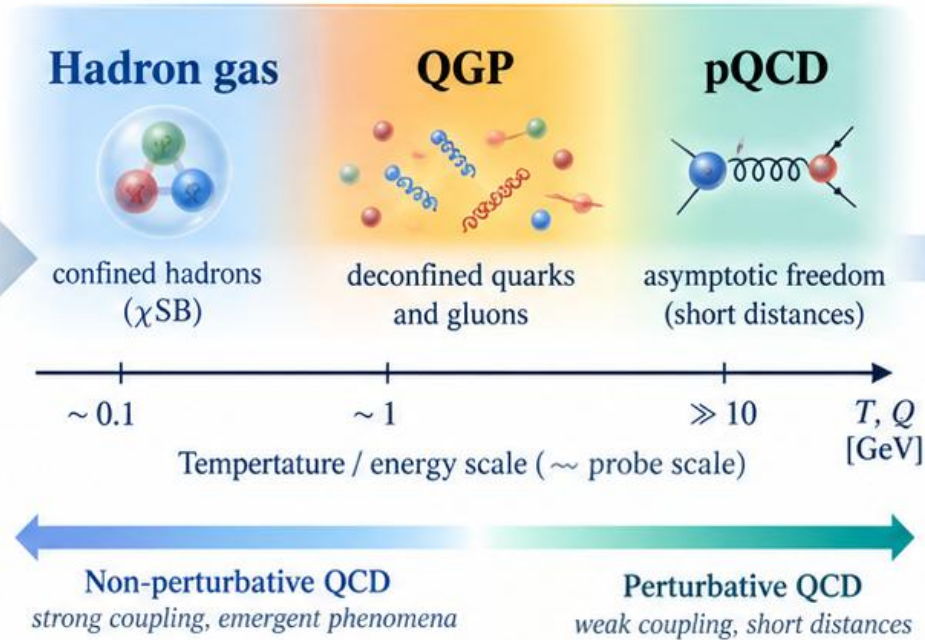
A colloquium-style bridge to perturbative calculations at colliders

Rotating QCD Matter in Strong Magnetic Fields

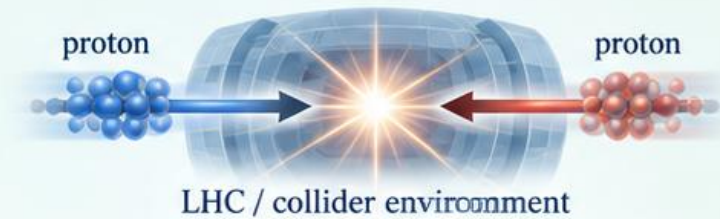


The Regimes of QCD

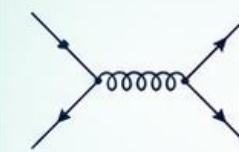
A unified theory across scales



High-Energy QCD at Colliders



Perturbative calculations

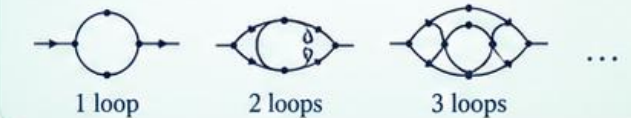


$$d\sigma = \sum |\mathcal{M}|^2 d\Phi$$

$$\mathcal{M} = \sum \alpha_s^n \mathcal{M}^{(n)}$$

$$\alpha_s(Q) \ll 1$$

Multi-loop amplitudes

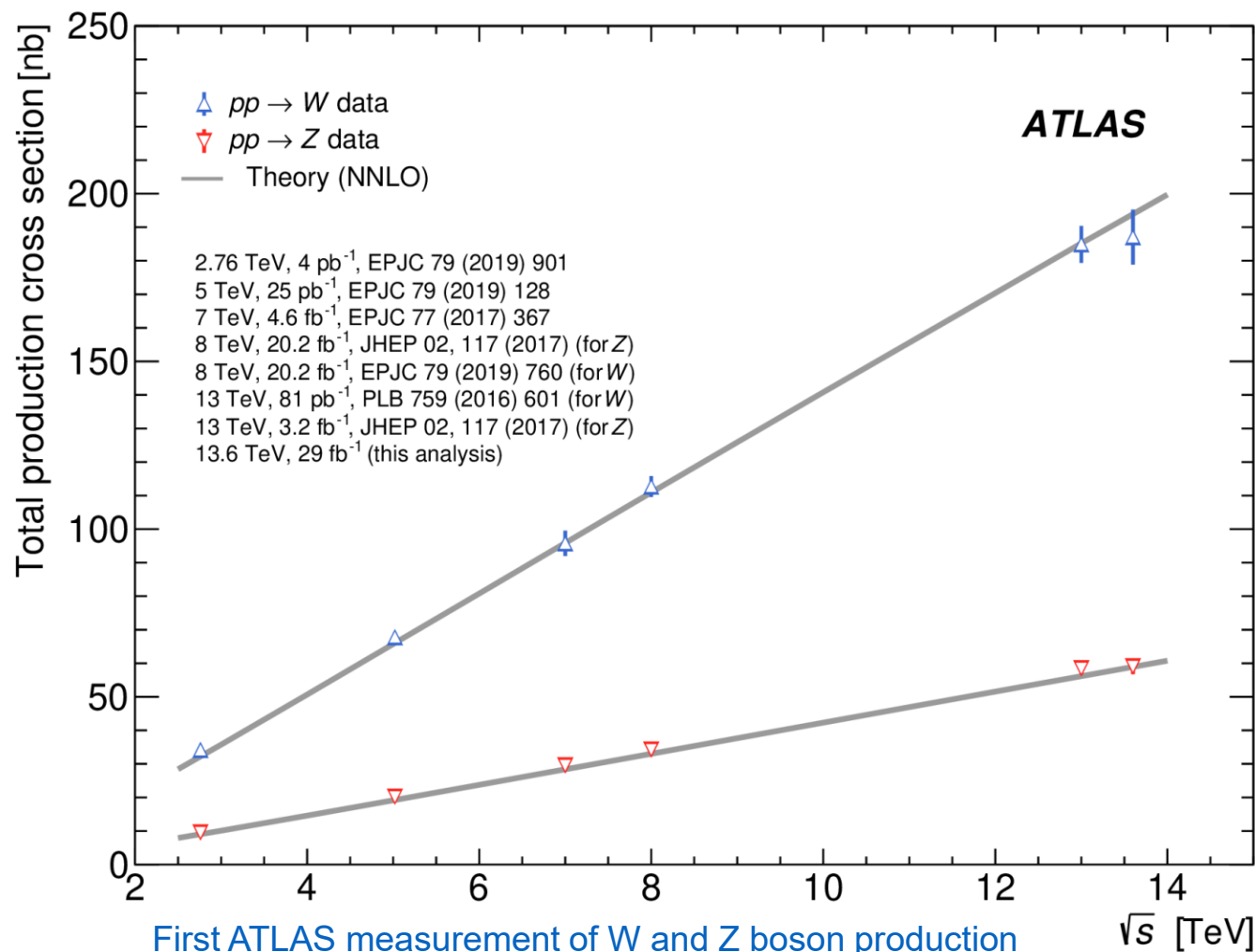


precision collider physics

Same theory, different regimes:

this talk focuses on the short-distance, perturbative side of QCD.

LHC measurements track Standard Model theory



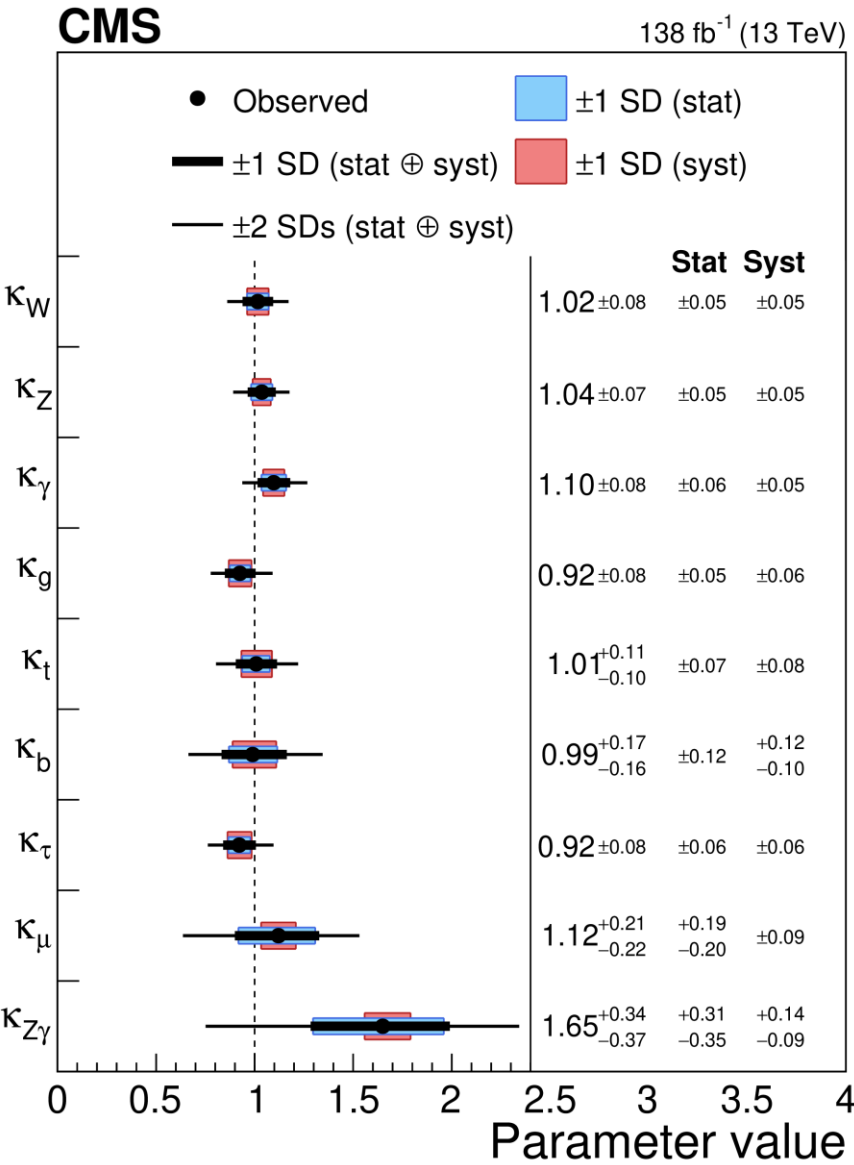
[First ATLAS measurement of W and Z boson production using Run-3 data | ATLAS Experiment at CERN](#)

The markers repeatedly land on the theory bands—across processes and collision energies.

Agreement over 13 orders of magnitude in production rates

pp, W/Z, top, Higgs, [ATLAS document](#)
dibosons, tribosons, four-top

Current data vs. theory



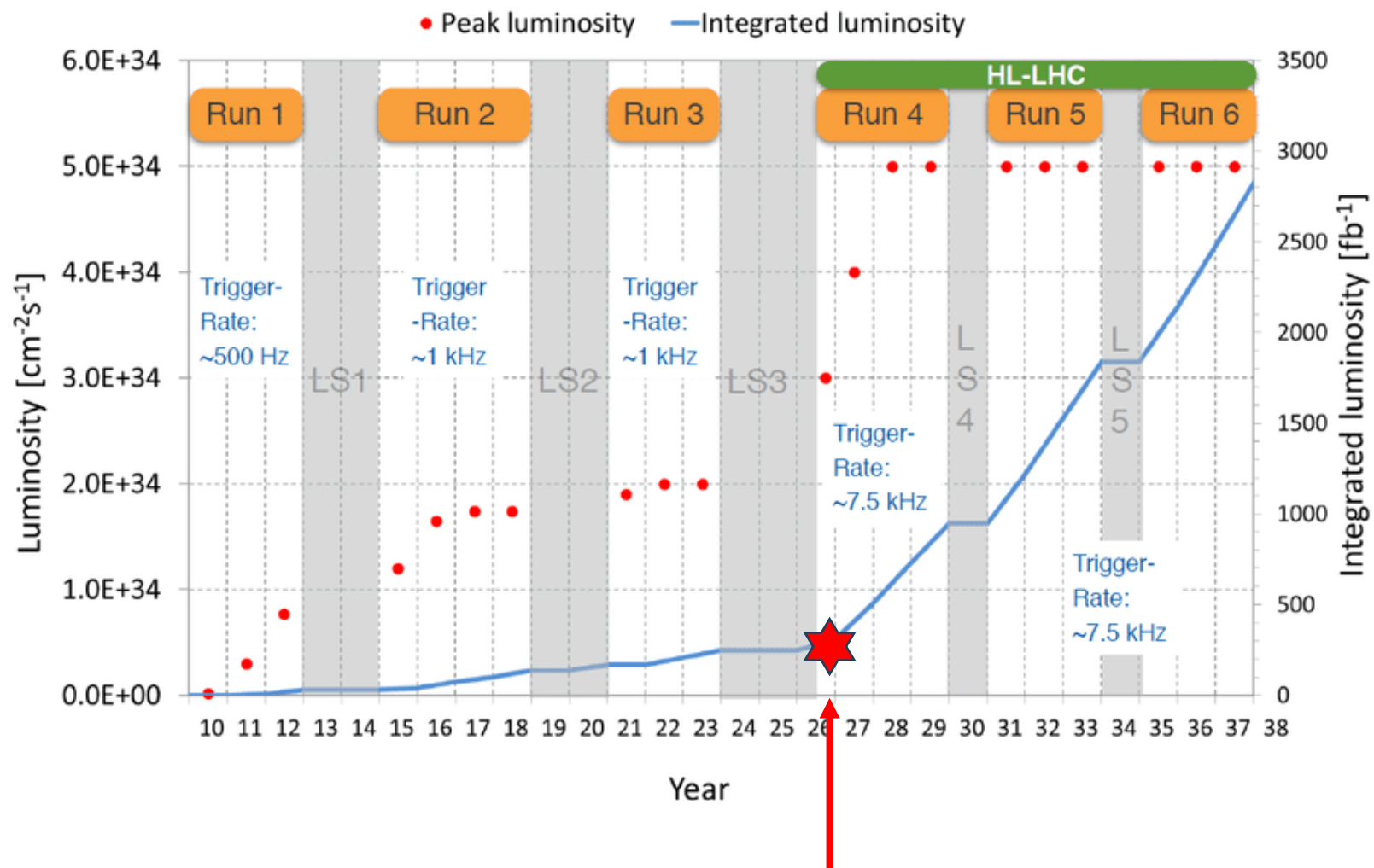
Coupling modifier $\kappa_X = \frac{y_X}{y_X^{\text{SM}}}$

Consistency check of the standard model

Today: 10% -- 20% uncertainty

[CMS-HIG-22-001](#)

High-luminosity LHC

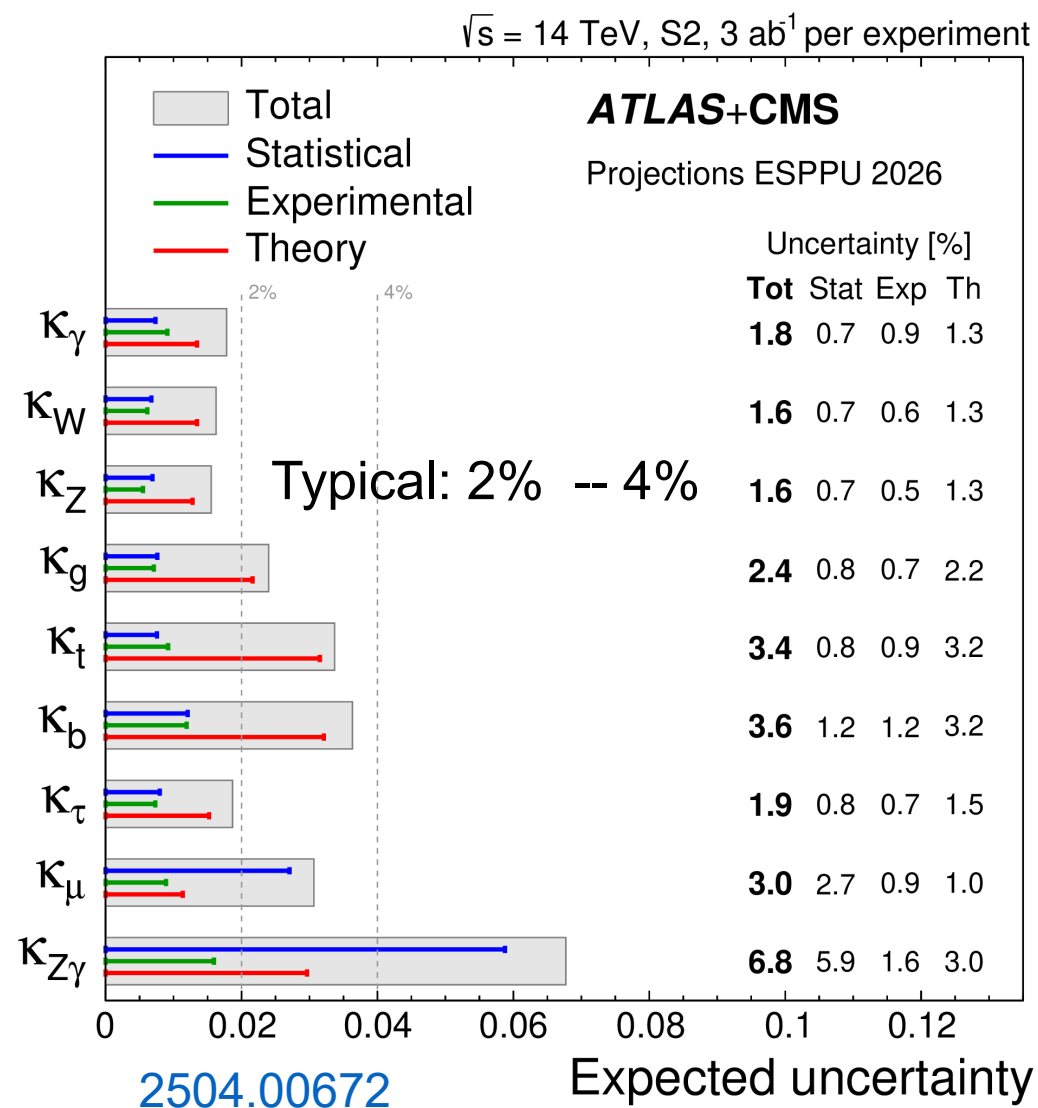


We are here

Luminosity: how many collision opportunities.

HL-LHC will deliver roughly **10x** more data in the next decade.

HL-LHC demands high-precision predictions



More data changes the bottleneck

1. Statistics improve

HL-LHC will record far more Higgs events, reducing statistical uncertainty

2. Theory becomes comparable

For several Higgs couplings, theory contributes as much uncertainty as the experimental components.

If theory does not improve, it—not the detector—sets the precision ceiling.

How precision predictions are built?

➤ How are precision predictions built?

- Perturbative expansion → Feynman diagrams → Feynman integrals

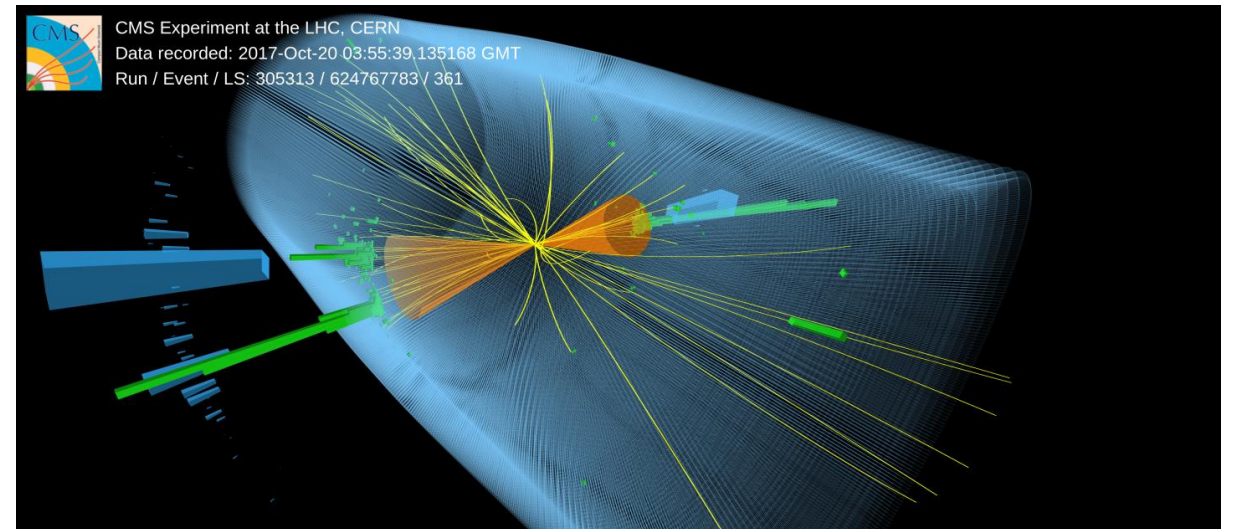
➤ What is the bottleneck?

- Reducing and evaluating complicated multiloop integrals

➤ A concrete example:

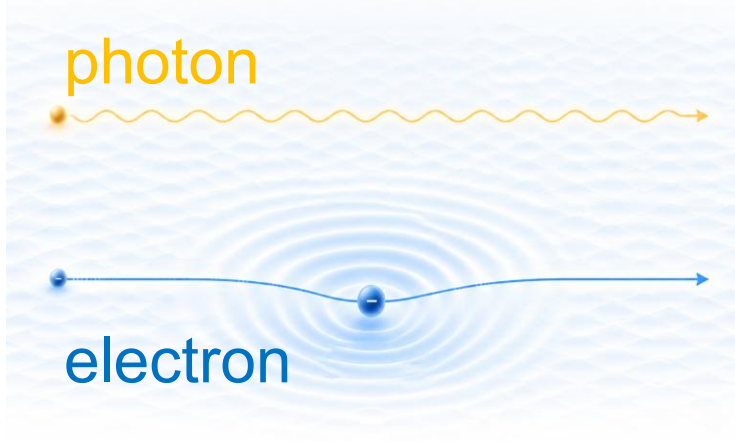
- Higgs + jet production

Jet: a narrow cone of particles



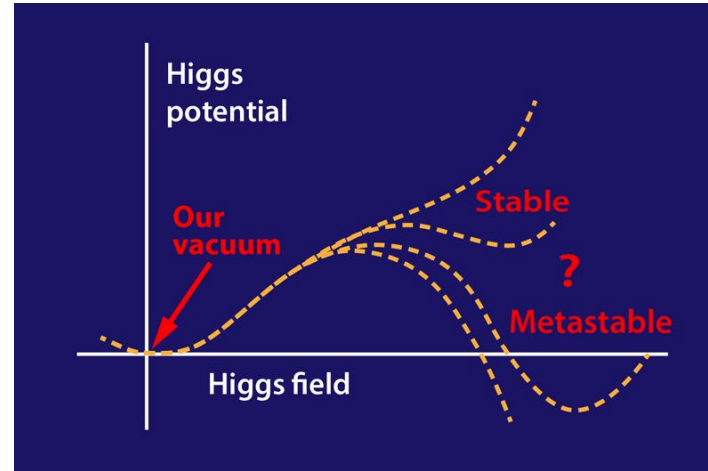
$$H(b\bar{b}) + j$$

Why Higgs



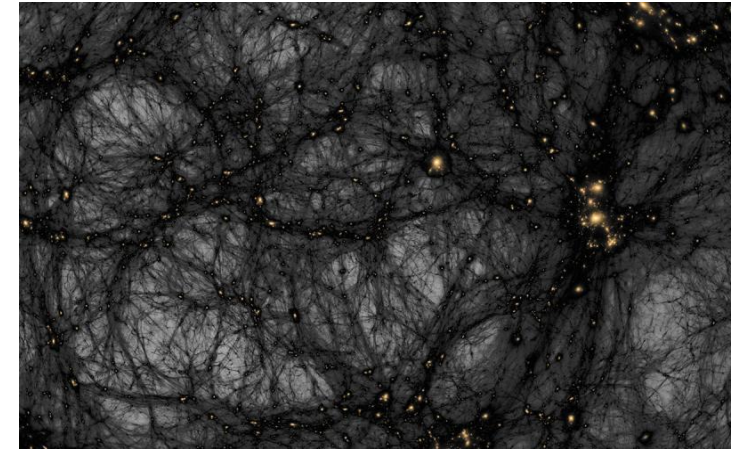
The origin of mass

Electroweak symmetry breaking



The stability of our universe

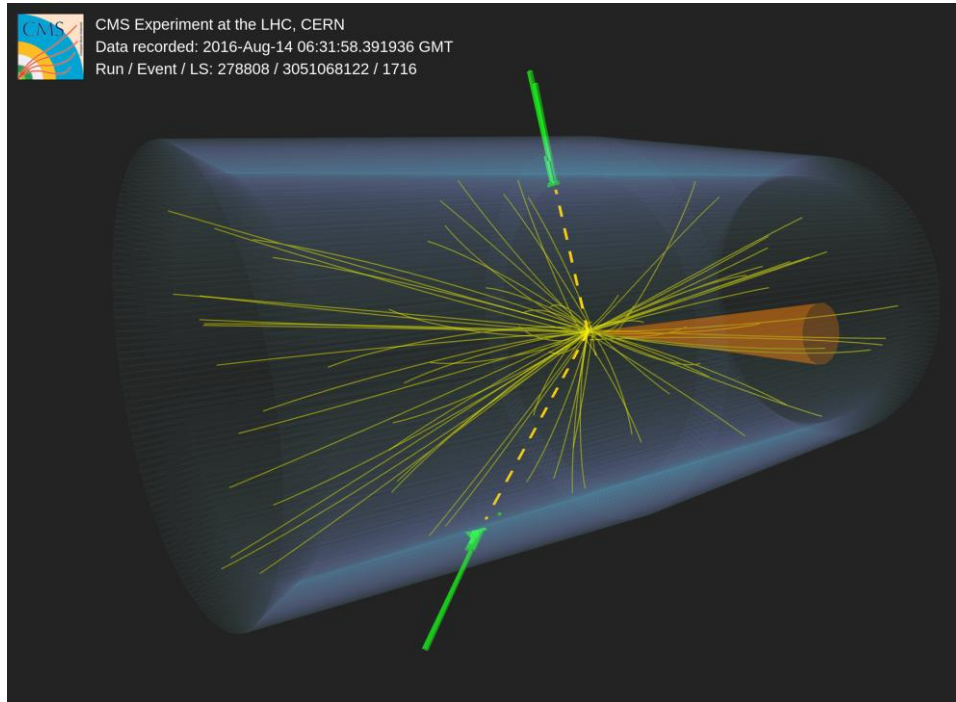
Higgs potential



Unique portal to new physics

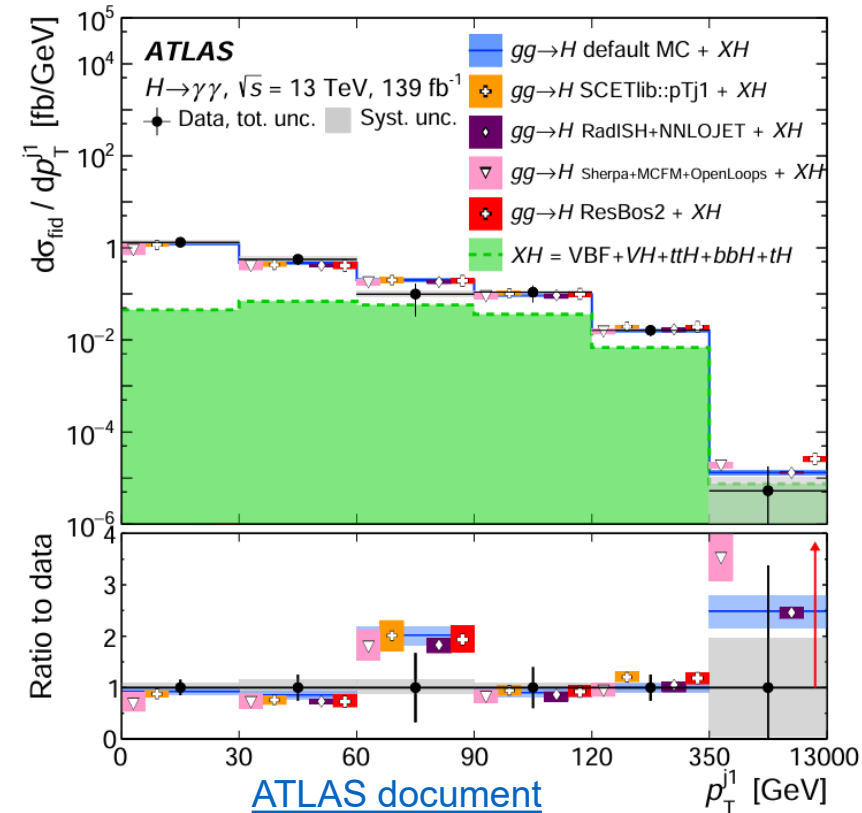
Composite Higgs?
Couples to new particles?

Higgs bosons are often produced with energetic jets



[CMS Experiment](#)

$H(\gamma\gamma) + j$



ATLAS/CMS measures the Higgs-plus-jet cross section over a wide range of jet transverse momenta.

Precise QCD predictions are needed to interpret this spectrum.

How many events do we produce?

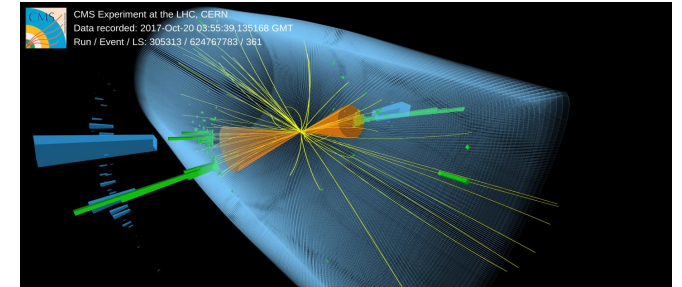
Expected number of Higgs + jet events

$$N_{H+J} \sim L_{\text{int}} \sigma(pp \rightarrow H + J)$$

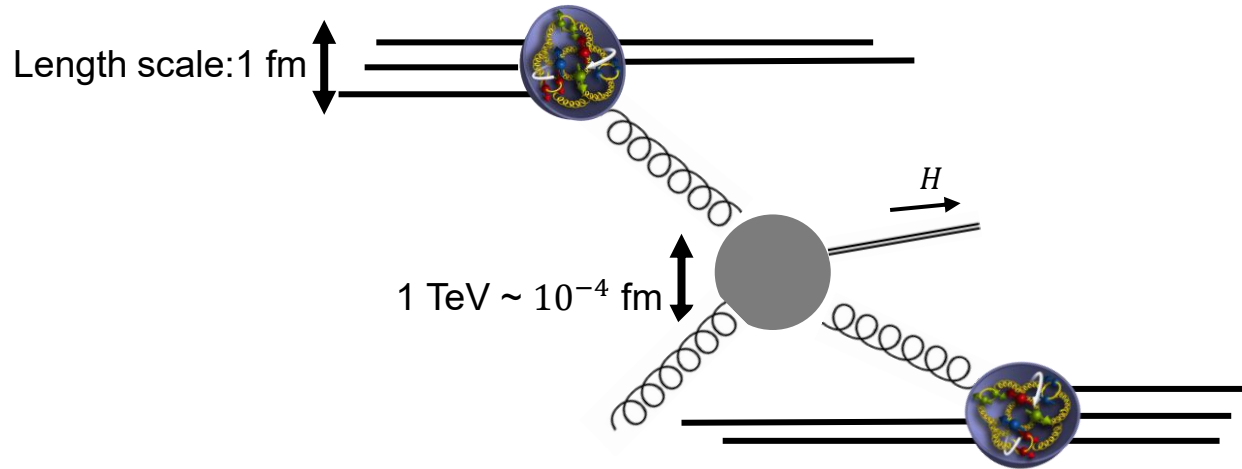
Luminosity: how many
collision opportunities.

Cross section:
How often the process happens.

Precise Higgs physics requires precise $\sigma(pp \rightarrow H + \text{jet})$.



From proton to parton



$$\sigma_{pp \rightarrow H+J} \sim f_{i \leftarrow p} \otimes f_{j \leftarrow p} \otimes \hat{\sigma}_{ij \rightarrow H+k}$$

Cross section: probability to produce a Higgs in **proton collisions**.

Probability to produce a Higgs in **parton collisions**.

Probability to find a parton

A parton (quark or gluon) forms a jet

Perturbative expansion

Expand the cross-section w.r.t the strong couple constant $\alpha_s \sim 0.1$.

$$\begin{aligned}\hat{\sigma} &= \hat{\sigma}^{(0)} && \text{Leading-order (LO)} \\ &+ \alpha_s^1 \hat{\sigma}^{(1)} && \text{Next-to-leading order (NLO)} \\ &+ \alpha_s^2 \hat{\sigma}^{(2)} && \text{NNLO} \\ &+ \alpha_s^3 \hat{\sigma}^{(3)} && \text{N3LO} \\ &+ \dots\end{aligned}$$

Each added order reduces missing-term uncertainty

Precision means computing high-order corrections!

From quantum amplitudes to measurable rates

1. Cross-section

$$\sigma \sim \int d\text{PS} |A|^2$$

Integrate over all possible final states

2. Amplitude

The quantum object sums over all possible paths

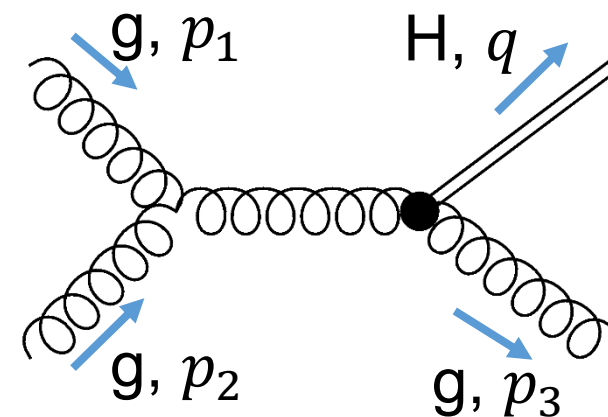
3. Feynman diagram

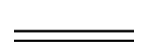
A visual bookkeeping language for the terms in amplitude.



A representative Higgs + jet diagram

Gluon fusion is the dominant production mechanism.



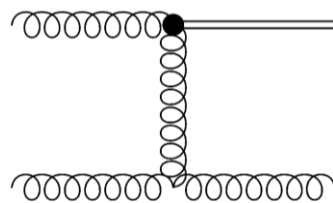
 gluon  Higgs

 ggH effective coupling

Momentum conservation: $p_1 + p_2 = q + p_3$

High-order corrections to the amplitude

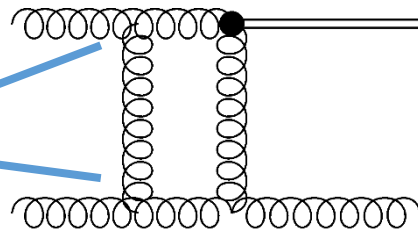
$$A = A^{(0)}$$



+ 3 diagrams

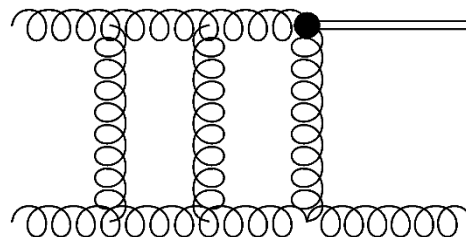
$$+ \alpha_S^1 A^{(1)}$$

α_S^1



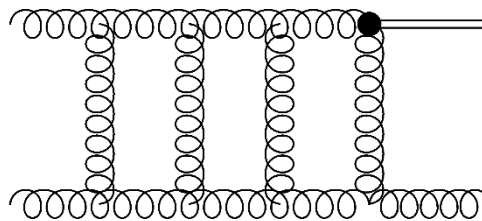
+ O(10) diagrams

$$+ \alpha_S^2 A^{(2)}$$



+ O(1000) diagrams

$$+ \alpha_S^3 A^{(3)} \\ + \dots$$

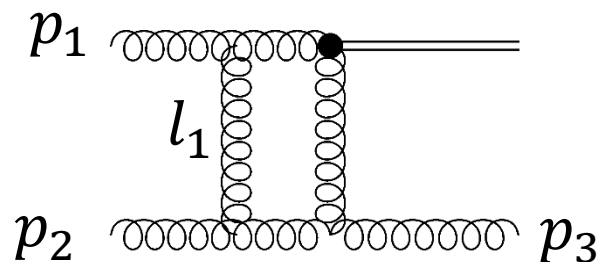


+ O(10,000) diagrams

Complexity blows up.

Feynman integrals

Amplitude can be written as a linear combination of Feynman integrals



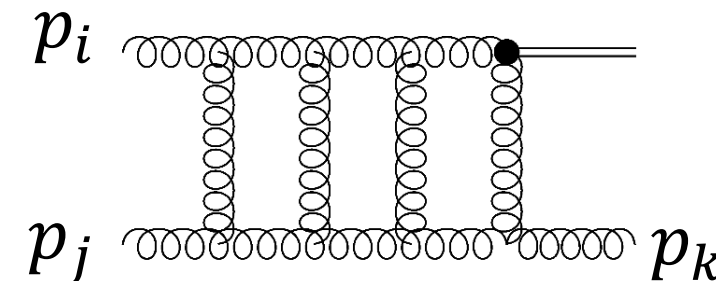
$$\sim \int_{-\infty}^{+\infty} d^d l_1 \frac{l_1 \cdot p_1}{l_1^2 (l_1 + p_1)^2 (l_1 - p_2)^2 (l_1 - p_2 + p_3)^2} + \dots$$

A blue arrow points from the text "Integrate over all kinematics of the intermediate particle" to the integration measure $d^d l_1$ in the equation.

Integrate over all kinematics
of the intermediate particle

Three-loop H+J amplitudes

$$A^{(3)} = \sum_{i=1}^{O(10^4)} f_i \times I_i$$



43183 Feynman diagrams,
6 million integrals
50 GB file size

After symmetry
→

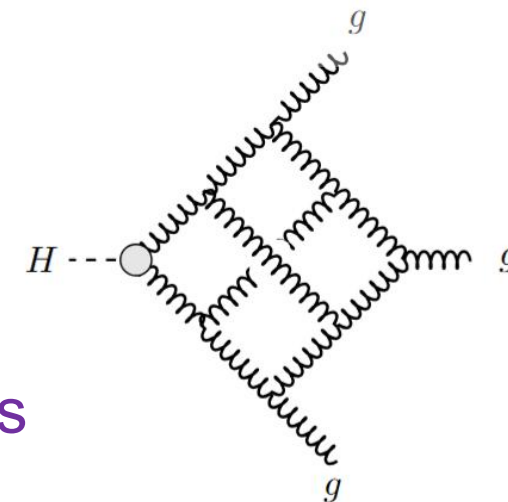
46 topologies,
200k integrals

Qgraf: Nogueira et al., Comput. Phys. 1993

CalcLoop: Ma et al., <https://gitlab.com/multiloop-pku/calclloop>

Bottleneck: Integral evaluation

Feynman integral is the main bottleneck:



15 indices: 10 propagators and 5 irreducible scalar products

$$\int \frac{d^d l_1 d^d l_2 d^d l_3 [(l_2 + p_1)^2]^{-a_{11}} [(l_3 + p_1)^2]^{-a_{12}} [(l_1 + p_2)^2]^{-a_{13}} [(l_3 + p_2)^2]^{-a_{14}} [(l_1 - p_3)^2]^{-a_{15}}}{[l_1^2]^{a_1} [(l_1 + p_1)^2]^{a_2} [l_2^2]^{a_3} [(l_2 + p_2)^2]^{a_4} [l_3^2]^{a_5} [(l_3 + p_3)^2]^{a_6} [-l_1 + l_3 + p_3]^2]^{a_7} [(l_2 + l_3)^2]^{a_8} [-l_1 + l_2 + l_3 - p_1]^2]^{a_9} [-l_1 + l_2 + l_3 + p_2 + p_3]^2]^{a_{10}}}$$

Encounter up to 6 numerator powers

The fundamental issue is not the millions of integrals, it's the complexity.

Linear relations among Feynman integrals

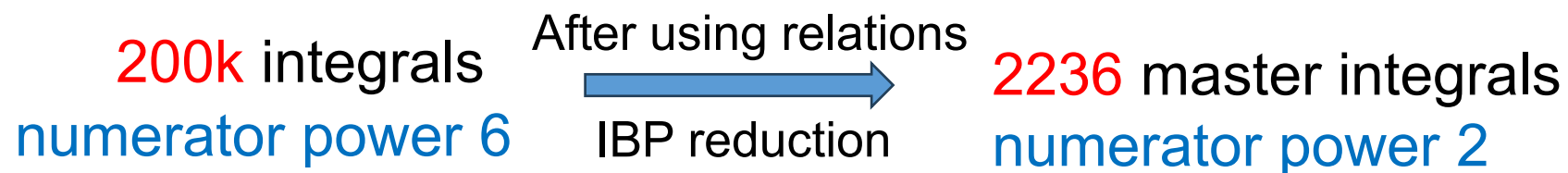
$$c_1 I_1 + c_2 I_2 + \cdots + c_n I_n = 0$$

Integration-by-part (IBP) identities

....

c_i are rational function coefficients

I_i are Feynman integrals



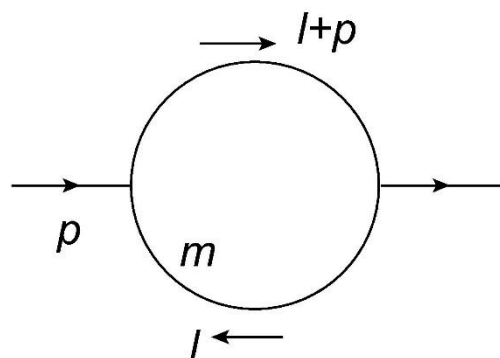
Fewer & simpler integrals to compute!

How to use Integration-by-part (IBP) identities?

Laporta algorithm

1. Target: $I(1, -2)$

2. Sow seeds (numerical $\vec{a}$), reap equations $I(1,0), I(1,-1), I(1,-2)$



$I(a_1, a_2)$

$$0 = \int \frac{d^d l}{i \pi^{d/2}} \frac{\partial}{\partial l^\mu} \frac{l^\mu}{(l^2 - m^2)^{a_1} ((l+p)^2 - m^2)^{a_2}}$$

$$0 = \int \frac{d^d l}{i \pi^{d/2}} \frac{\partial}{\partial l^\mu} \frac{p^\mu}{(l^2 - m^2)^{a_1} ((l+p)^2 - m^2)^{a_2}}$$



IBP identity

$$\begin{aligned} (2 - 2\epsilon) I[1, 0] - 2m^2 I[2, 0] &= 0 \\ -I[1, 0] + I[2, -1] - s I[2, 0] &= 0 \\ (3 - 2\epsilon) I[1, -1] + (2m^2 - s) I[1, 0] - 2m^2 I[2, -1] &= 0 \\ -2I[1, -1] - s I[1, 0] + I[2, -2] - s I[2, -1] &= 0 \\ (4 - 2\epsilon) I[1, -2] + (4m^2 - 2s) I[1, -1] - 2m^2 I[2, -2] &= 0 \\ -3I[1, -2] - 2s I[1, -1] + I[2, -3] - s I[2, -2] &= 0 \end{aligned}$$

3. Gaussian elimination

$$I[1, -2] \rightarrow \frac{s(-2m^2 - 2s + s\epsilon) I[1, 0]}{-2 + \epsilon}$$

Master
integral

Simpler!

Challenges in IBP reduction

Complicated intermediate expression

Simple input => intermediate expression swell => (relatively) simple output

Resource-consuming due to large scale of linear equations

e.g. 5×10^8 equations in four-loop
QED form factors

Laporta, 1910.01248

O(TB) RAM

e.g. V+J (similar to H+J, but simpler)

Mella, Pos LL2024

Months of runtime

e.g. Five-loop QCD beta function uses
 $O(10^6)$ core*h.

Baikov, Chetyrkin, Kuhn, 1606.08659



Selected developments in IBP reduction

Syzygy

Eliminate unnecessary integrals in the equation,
delete duplicate computation

Gluza, Kajda, Kosower, 1009.0472

NeatIBP, Wu, Boehm, Ma et al., 2305.08783

Finite Field

Solve the linear system in finite field (**numerical sampling**)
and reconstruct the analytic results.

Manteuffel, Schabinger, 1406.4513

FiniteFlow, Peraro, 1905.08019

Firefly, Klappert, Lange, 1904.00009

Spanning-cut/sector

Divide the linear system into subsystems,
divide and conquer.

Larsen, Zhang, et. al., 1511.01071,
1805.01873, 2104.06866

Blade, XG, Liu, Ma, Wu. 2405.14621

Block-triangular form

Recognize and exploit block
structures in the relations.

Liu, Ma, 1801.10523

XG, Liu, Ma, 1912.09294

Blade, XG, Liu, Ma, Wu. 2405.14621

Improved seeding

Avoid generating unnecessary
seeds (relations) from the beginning.

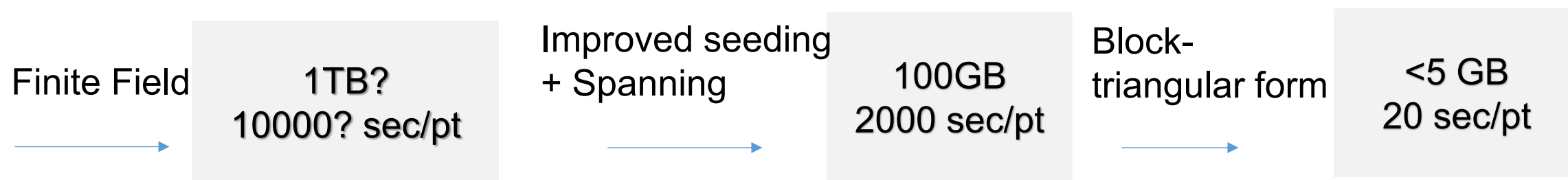
Blade, XG, Liu, Ma et al., [2405.14621](#)

Driesse, Jakobsen, Mogull et al., [2403.07781](#)

Bern, Herrmann, Roiban et al., [2406.01554](#)

Make the system smaller and the solving faster!

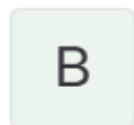
IBP reduction summary



We made the reduction feasible on realistic computing resources with Blade.

- Reduce memory usage significantly!
- Orders-of-magnitude speedup.

Several hundreds of thousands core*hours



Blade 

Link: <https://gitee.com/multiloop-pku/blade>

项目信息

Block-triangular form improved Feynman integral decomposition .

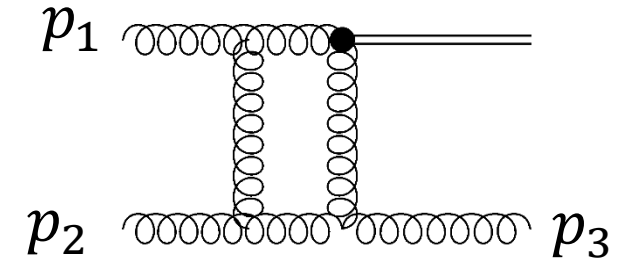
Blade: XG, Liu, Ma, Wu. 2405.14621

Compute master integrals using differential equations

Use IBP to construct differential equations of master integrals

$$\frac{\partial I[1,1,1,1]}{\partial t} = -\frac{(s+t)}{2t} I[1,2,1,1] + \dots$$

$$\text{IBP} = \frac{s+t}{s t} I[1,1,1,1] + \dots$$



$$\frac{\partial}{\partial t} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = B_t(t, u, \epsilon) \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} \quad B_t = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \# & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \# & \# & \# & \# \end{pmatrix}$$

$$\begin{aligned} t &= (p_1 - p_3)^2 \\ u &= (p_2 - p_3)^2 \\ d &= 4 - 2\epsilon \end{aligned}$$

Same for $B_u(t, u, \epsilon)$

Solve DEs analytically or using series expansion

Canonical differential equation makes the analytic solution transparent

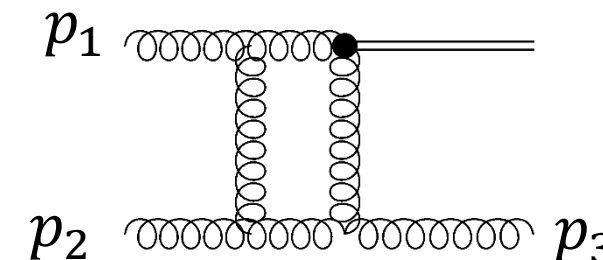
Find a basis transformation $\vec{I} = T \vec{f}$

such that ϵ is factorized

$$d \vec{f} = \epsilon \sum_k A_k d\text{Log}(\alpha_k) \vec{f}$$

Solve DE in terms of Lauren expansion ϵ

$$\begin{aligned} f_4 = & 1 - \epsilon [\log(1 - t - u) + \log(u)] \\ & + \epsilon^2 \left[\frac{1}{2} \log^2(1 - t - u) + \frac{1}{2} \log^2(u) + \log(1 - t - u) \log(u) + \text{Li}_2(t + u) + \text{Li}_2(1 - u) - \frac{\pi^2}{6} \right] \\ & + O(\epsilon^3) \end{aligned}$$



$$d = 4 - 2\epsilon$$

Introducing ϵ is a calculation trick. We only need first a few orders.

H+J master integrals

We invested substantial effort in transforming the differential equation into canonical form.

The canonical form is extremely simple

4 GB DE -> 5 MB Canonical DE

New structures arise at three-loop level

Results

Compact analytic expressions in Multiple Polylogarithms, through ϵ^6

Lee, 1411.0911; Meyer, 1611.01087, 1705.06252

Gehrmann et al., 2410.19088

Di Vita et al., 1408.3107

Canko et al., 2112.14275

Henn et al., 2302.12776

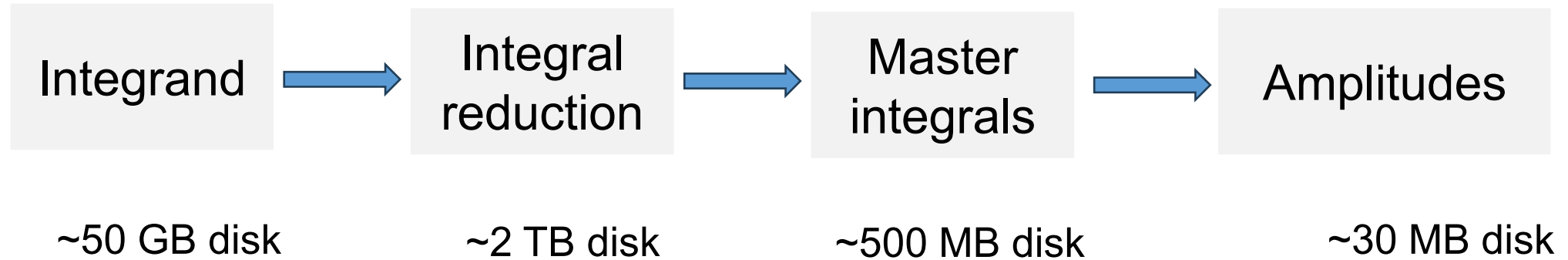
Syrrakos et al., 2307.08432

XG, Mistlberger, Ruf, [2511.13845](#)

XG, Mistlberger, Ruf, 2607.12033

$$d\vec{f} = \epsilon \sum_k A_k d\text{Log}(\alpha_k) \vec{f}$$

Assemble the amplitude



Analytic form in terms of multiple polylogarithms

Validation

Numerical Check • Collinear Limit • UV&IR Structure • N=4 SYM Structure

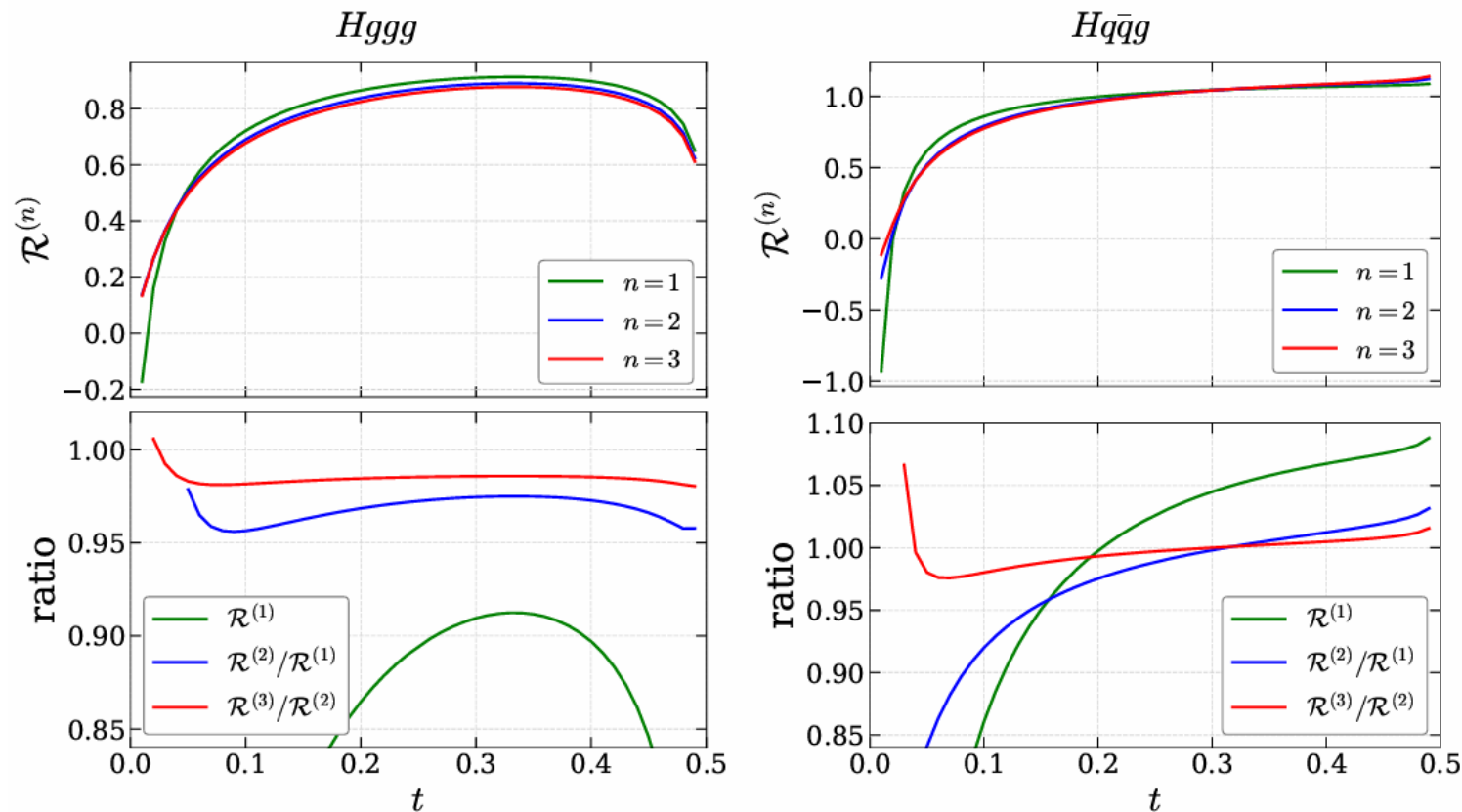
AMFlow: Liu, Ma. [2201.11669](#)

XG, Herzog, Ma, Mistlberger,
Suresh, [2408.03019](#)

Almelid, Duhr, Gardi, [1507.00047](#)
Becher, Neubert, [1908.11379](#)

Dixon et al., [2012.12286](#); [2204.11901](#)
Gehrmann et al., [2410.19088](#)
XG, Mistlberger, Ruf, [2511.13845](#)

Numerical results of the amplitudes



Good convergence behavior

~2% corrections at three-loop
(not a physical prediction yet)

Easy to evaluate

ready for phenomenology
applications

Numerical results of interfered amplitudes at different perturbative order, along the $t = u$ line.

One H + jet amplitude unlocks three Higgs predictions

LHC

$N^3\text{LO}$ Higgs + jet observables

Combined with the remaining real-radiation ingredients.

Future e^+e^- colliders

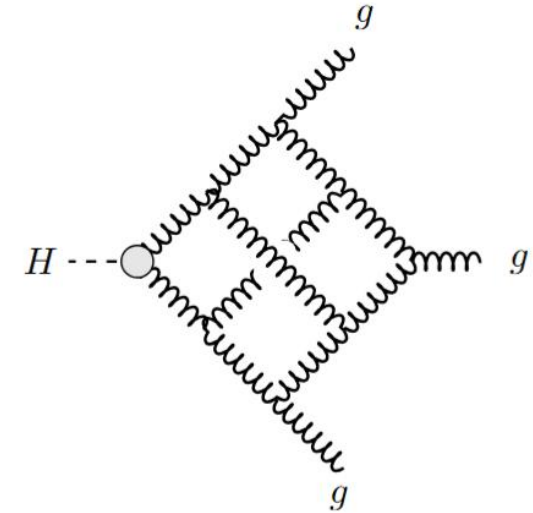
$N^3\text{LO}$ $H \rightarrow 3$ jets

The same amplitude is reused through crossing symmetry.

Inclusive Higgs production

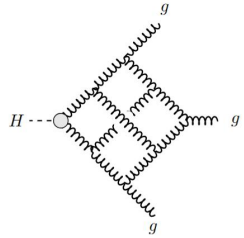
A building block for $N^4\text{LO}$

Integrating over final states feeds the total rate.



One three-loop object connects differential, decay, and inclusive precision.

The calculation unlocks a reusable three-loop toolkit



1 massive leg • $2 \rightarrow 2$ scattering • 3 loops

Phenomenology

$b \bar{b} \rightarrow Hg$ • $W/Z + \text{jet}$

T. Gehrmann, P. Jakubčík, C. C. Mella et al., 2307.15405

XG, Mistlberger, Ruf, To appear

Reuse the integral basis in other one-mass processes.

Amplitude theory

$N = 4$ SYM form factors

Gehrmann et al., 2410.19088

XG, Mistlberger, Ruf, 2511.13845

Inputs for bootstrap programs and analytic structure.

Next frontier

Four-loop / $N^4\text{LO}$ Higgs production

Lee et al. [2202.04660](#)

Mistlberger, Suresh, [2504.10574](#)

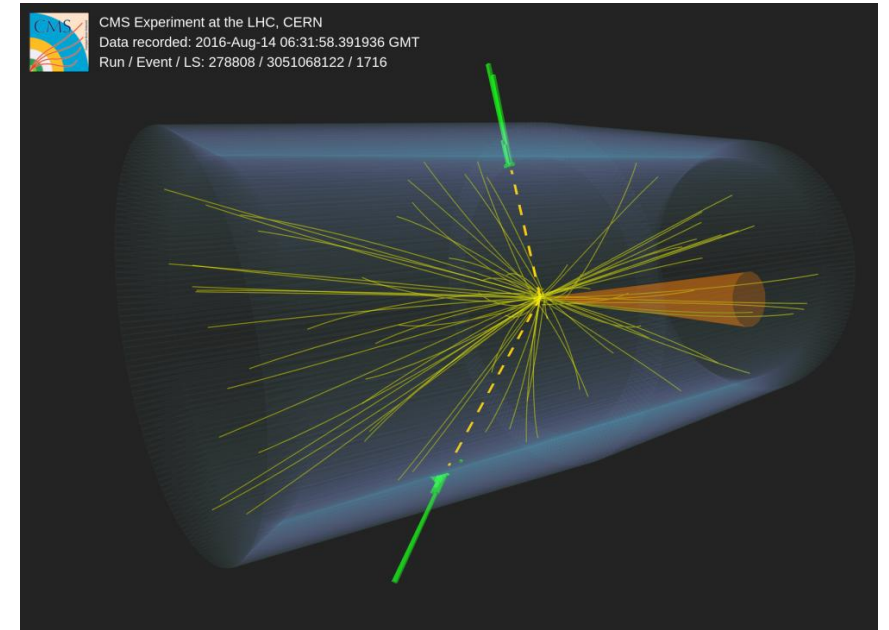
The same techniques make the next order approachable.

The deliverable is not only one amplitude—it is reusable mathematical infrastructure.

The public codes, e.g. Blade, will benefit the broader community.

Conclusions

- High-energy QCD provides a complementary regime of the same theory.
- Computational complexity is the main challenge in higher-order calculations.
- Three-loop H + jet is both an essential ingredient and a methods benchmark.



$$p p \rightarrow H(\gamma\gamma) + j$$

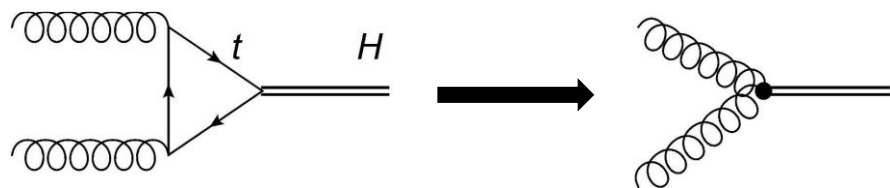
Looking ahead:

Make N3LO H+J production and N4LO inclusive production feasible.

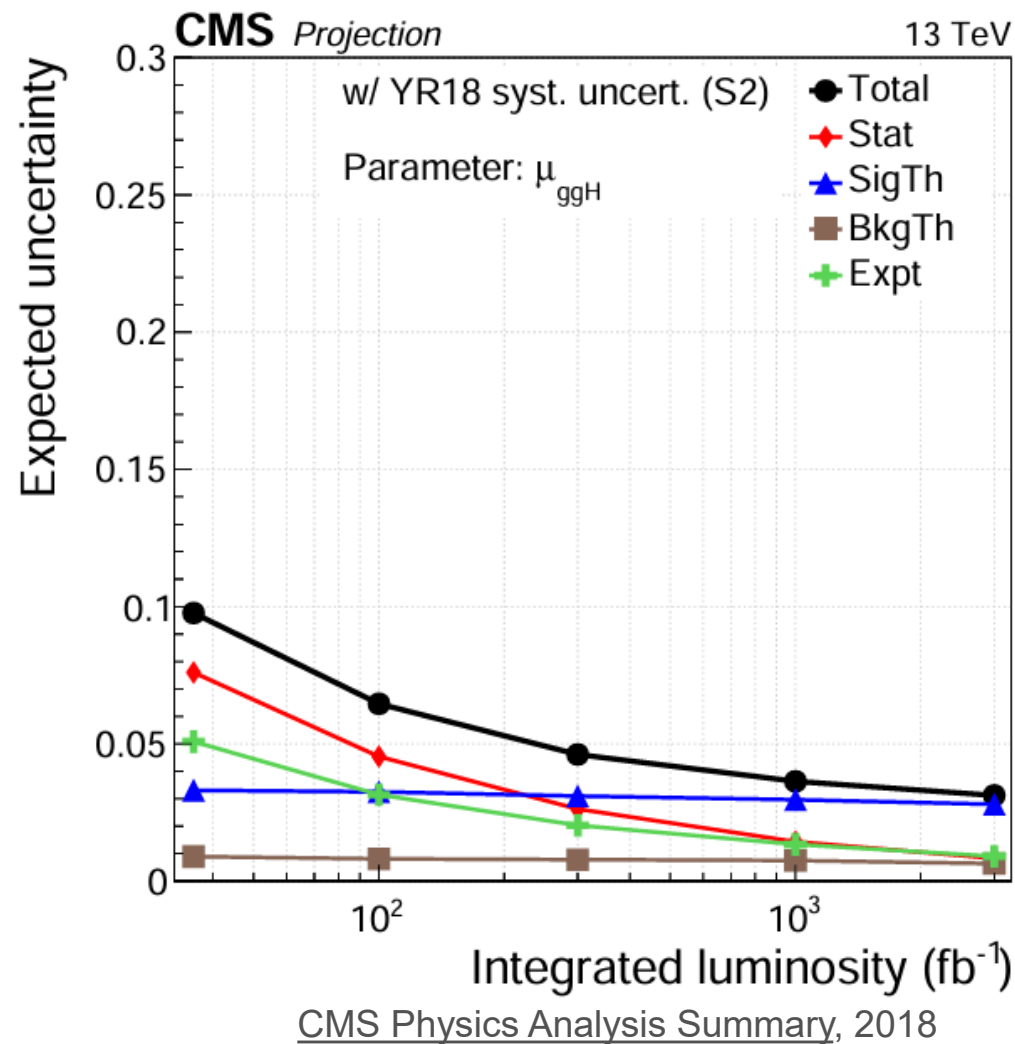
Thank you!

N4LO Higgs production

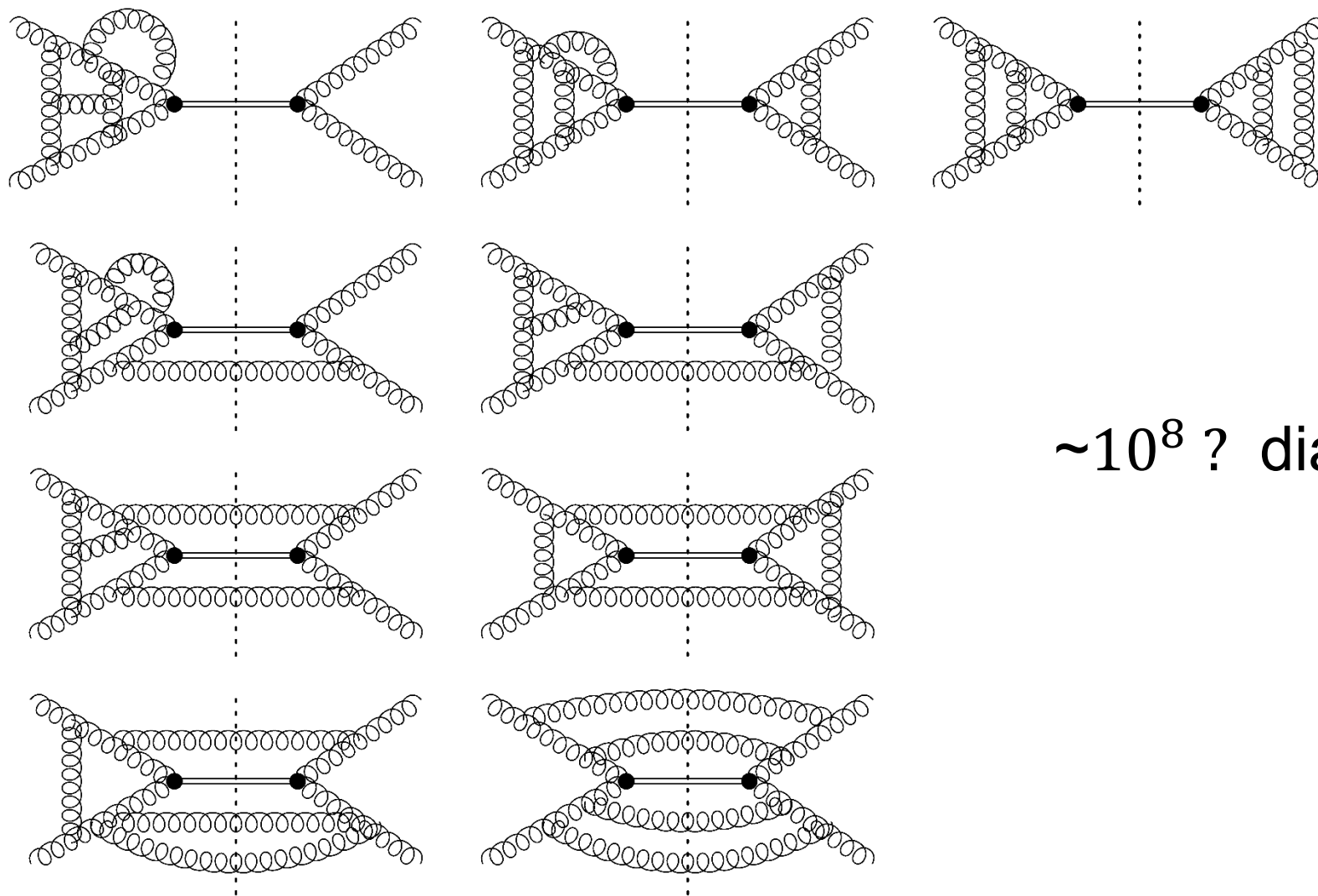
- Effective Higgs coupling to gluons



- Percent-level precision at HL-LHC
- Achieving the expected theoretic uncertainty is already highly nontrivial!



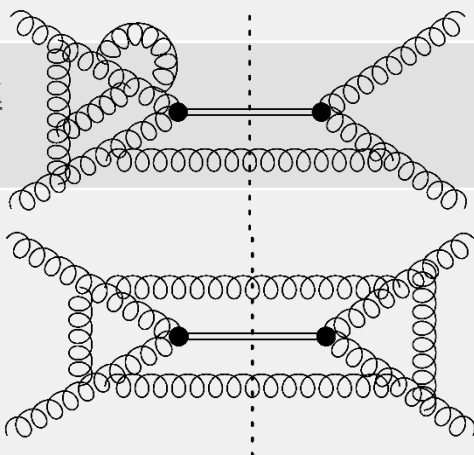
N4LO Higgs production



$\sim 10^8$? diagrams

N4LO Higgs production

Compute building blocks and then assemble. **Virtual**: loop corrections.
Real: extra emission particles.

# Real	Block	Status	Feynman diagram
0 real	VVVV	Known Lee, Manteuffel, Schabinger et al. 2202.04660	
1 real	RVVxV	Known Mistlberger, Suresh, 2504.10574	
	RVVV	To appear (based on the H+J amplitude) Mistlberger, Suresh, LoopFest2026 Talk	
2 real	RRVxV	To appear strict-threshold limit computed Banik, XG, Mistlberger, Ruf, Suresh	
	RRVV	In progress	
3 real	RRRV	In progress Banik, XG, Mistlberger, Ruf, Suresh	
4 real	RRRR	In progress	

Integral evaluation

Feynman integral remains the main bottleneck

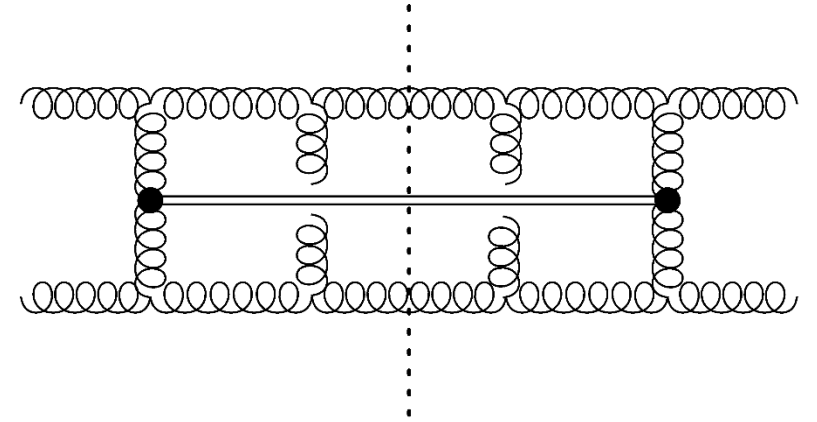
Feynman integral simplifies in the threshold/soft limit

The most complicated master integral:

10 denominators + 3 delta functions + numerator power 1

$$\int \frac{d^d p_4 d^d p_5 d^d p_6 d^d p_7 \delta[(p_2 + p_3) \cdot (p_2 + p_3 + 2p_4 + 2p_5)] \delta(p_4^2) \delta(p_5^2) (p_6 + p_7)^2}{[2p_3 \cdot (p_4 - p_6)] (p_4 - p_6)^2 p_6^2 [2p_2 \cdot (p_5 + p_6)] (p_5 + p_6)^2 [2p_2 \cdot (p_4 - p_7)] (p_4 - p_7)^2 p_7^2 [2p_3 \cdot (p_5 + p_7)] (p_5 + p_7)^2}$$

$$\sim -\frac{37}{15} \epsilon^{-7} + \frac{128}{9} \epsilon^{-6} + \left(\frac{1006}{45} + \frac{529 \pi^2}{30} \right) \epsilon^{-5} \\ + \frac{-22718 + 3754 \pi^2 - 14242 \text{Zeta}[3]}{45} \epsilon^{-4} + \dots + O(\epsilon^2)$$



$$d = 4 - 2\epsilon$$

Multiple zeta values

Computed with optimized
auxiliary mass flow method

Difficulty of IBP reduction: **highly coupled • auxiliary integrals**

Key idea:

1. Strict block upper-triangular form.

Solve by blocks

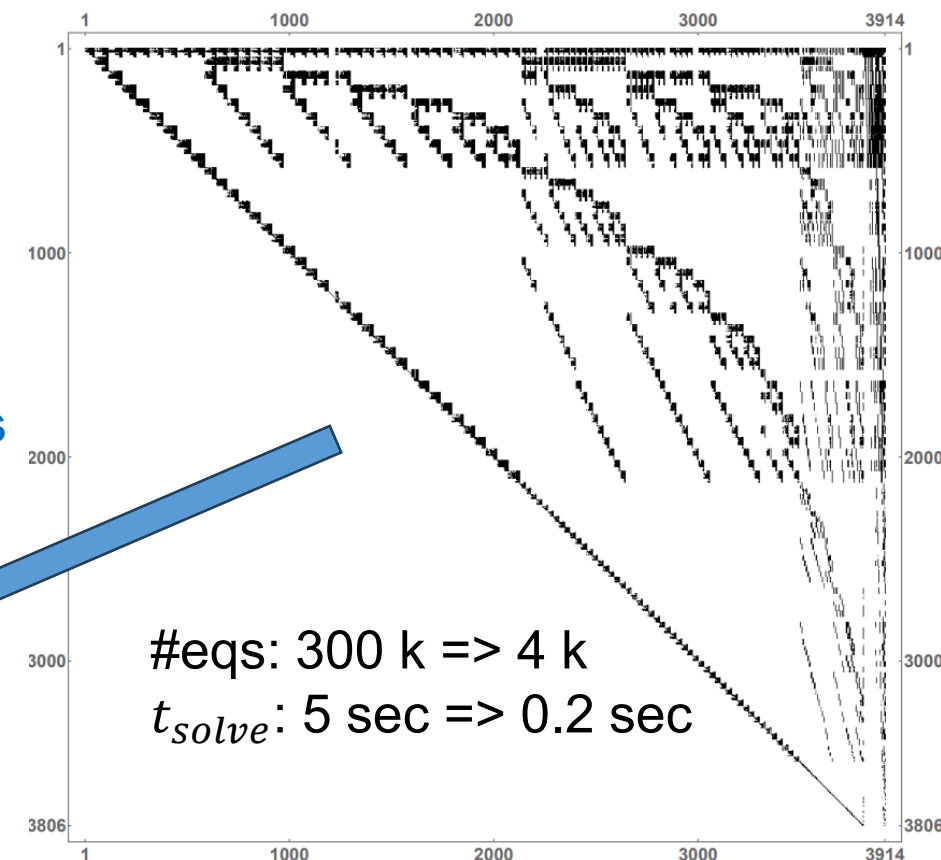
2. Search/fit relations among needed integrals

Orders of magnitude fewer equations

$$\begin{aligned} t^2 I[1,1,-2] + u^2 t I[1,1,-1] + \epsilon t^4 I[1,1,0] + u^2 I[0,1,-1] &= 0 \\ u^2 I[1,1,-2] + t^2 u I[1,1,-1] + \epsilon^2 (u^2 t^2 + t^4) I[1,1,0] &= 0 \end{aligned}$$

Each row is an equation

Each column represents an integral $I[\vec{a}]$

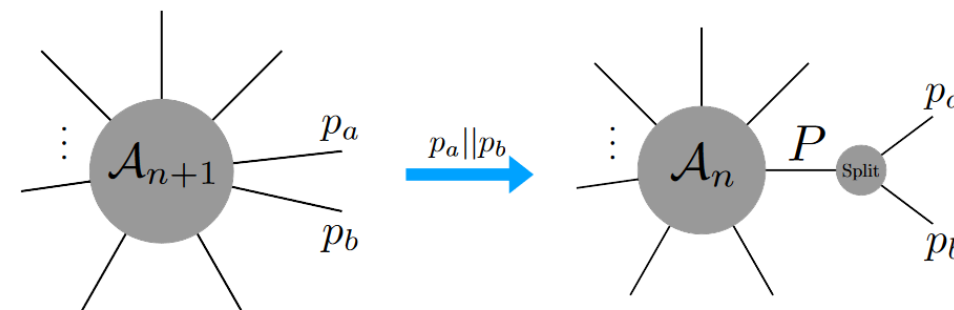


An example of block-triangular form

Validation: Collinear limit

1. A universal limit provides a powerful check.

$$A_{Hggg} \rightarrow \text{Split} \times A_{Hgg}$$



XG Herzog, Ma, Mistlberger, Suresh,
2408.03019

The amplitude factorizes.

1. Two particles moving in the same direction (collinear).
2. Other particles cannot resolve the inner structure of the two collinear particles

The splitting amplitudes are universal.

Validation: structural and numerical checks

2. UV and IR divergence (ϵ poles) match universal predictions.

Almelid, Duhr, Gardi, [1507.00047](#)

Becher, Neubert, [1908.11379](#)

3. Independent numerical checks at one phase space point

$$\left(t = \frac{1}{3}, u = \frac{1}{3}\right).$$

AMFlow: Liu, Ma. [2201.11669](#)

4. Leading transcendental structure matches known analytic structures
($\mathcal{N}=4$ sYM)

Dixon et al., [2012.12286](#); [2204.11901](#)

Gehrmann et al., [2410.19088](#)

XG, Mistlberger, Ruf, [2511.13845](#)